



2017-2018 年度美国“大联盟”(Math League)思维探索第一阶段活动  
(七年级)

(活动日期: 2017 年 11 月 26 日, 答题时间: 90 分钟, 总分 200 分)

学生诚信协议: 答题期间, 我确定没有就所涉及的问题或结论, 与任何人、用任何方式交流或讨论, 我确定我所填写的答案均为我个人独立完成的成果, 否则愿接受本次成绩无效的处罚。

选择题: 每小题 5 分, 答对加 5 分, 答错不扣分, 共 200 分。

- Drawing the diagonals of a rectangle creates exactly ? triangles.  
A) 2                      B) 4                      C) 6                      D) 8
- The least possible average of 2017 different positive integers is  
A) 1008                  B) 1009                  C) 2017                  D) 2018
- There were seven friends who decide that they would all dine together every evening if they could sit in a different arrangement each time. They would use the same table, always with seven chairs in the same spots. (Two arrangements are considered identical if and only if everyone of the seven friends sits on the same chair.) How many dinners could the seven of them eat before exhausting all possible arrangements?  
A) 2520                  B) 5040                  C) 720                    D) 1440
- Increasing a number by 20% is the same as multiplying it by  
A) 20%                    B) 80%                    C) 120%                    D) 200%
- \$100 in nickels is ? more coins than \$100 in dimes.  
A) 100                    B) 200                    C) 1000                    D) 2000
- What is the range of any 2018 consecutive integers?  
A) 1009                  B) 2017                  C) 2018                  D) 2019
- Written as a decimal,  $\frac{123456789}{100}$  has exactly ? non-zero digits to the right of the decimal point.  
A) 2                      B) 3                      C) 6                      D) 7
- Each choir member sang 1 song alone and 2 songs with the entire choir. If 24 songs were sung in all, the choir must have ? members.  
A) 8                      B) 11                      C) 12                      D) 22
- A multiple of 2017 is divided by a multiple of 2018. What is the least remainder possible?  
A) 0                      B) 1                      C) 2017                    D) 2018
- My armful of identical gumballs weighs 4% less since I dropped one gumball. How many gumballs are in my arms now?  
A) 23                    B) 24                    C) 25                    D) 26



11. The digits of the least 2-digit integer that is a perfect square *and* a perfect cube have the sum  
 A) 7                      B) 8                      C) 9                      D) 10
12. The year in which my grandfather was born, a perfect square, when subtracted from the year in which my daughter was born, another perfect square, gives my grandfather's age when he died. If my grandfather had lived, I would have been exactly half his age in 1943. How old was I in 1943?  
 A) 42                      B) 44                      C) 46                      D) None of the above
13. A man had two horses. He sold one of them on Tuesday for \$198 and made a profit of ten percent. On Wednesday, he sold the other one for \$198 and took a loss of ten percent. Tallying up his two deals, did he show a net profit or a loss?  
 A) Even                      B) A net profit of \$6  
 C) A net profit of \$4                      D) A net loss of \$4
14. The sum of the lengths of all the edges of a cube is 144 cm. What is the area of one face of the cube?  
 A)  $144 \text{ cm}^2$                       B)  $196 \text{ cm}^2$                       C)  $256 \text{ cm}^2$                       D)  $324 \text{ cm}^2$
15. The time 815 minutes after 8:15 P.M. is  
 A) 3:15 A.M.                      B) 9:50 A.M.                      C) 3:15 P.M.                      D) 9:50 P.M.
16. The number 180 has ? more divisors than the number 120 has.  
 A) 0                      B) 2                      C) 30                      D) 60
17. The 8 houses on my street have consecutive integer addresses that add up to 1500. The address with the greatest numerical value is  
 A) 184                      B) 187                      C) 188                      D) 191
18. Which of these fractions is the sum of an integer and its reciprocal?  
 A)  $\frac{7}{3}$                       B)  $\frac{8}{3}$                       C)  $\frac{9}{3}$                       D)  $\frac{10}{3}$
19. The mixed number  $2\frac{1}{4}$  is equivalent to many improper fractions that have integer numerators and denominators. The numerator of such a fraction could be any of the following except  
 A) 24                      B) 27                      C) 36                      D) 45
20. At my store, \$1 of every \$5 in sales is profit. If I split 10% of all profits equally among 10 people, each gets ?% of the total sales.  
 A) 0.2                      B) 2                      C) 5                      D) 20
21. If Mary is twice as old as Ann was when Mary was as old as Ann is now, and Mary is 32, how old is Ann?  
 A) 20                      B) 24                      C) 32                      D) None of the above
22. Of the following, which expression has the least value?  
 A)  $\frac{3^{100}}{4}$                       B)  $\left(\frac{3}{4}\right)^{100}$                       C)  $\frac{3}{4}$                       D)  $\frac{3}{4^{100}}$
23. I randomly select a positive integer less than 100. The probability that it is the product of exactly 3 different primes is  
 A)  $\frac{1}{99}$                       B)  $\frac{4}{99}$                       C)  $\frac{5}{99}$                       D)  $\frac{8}{99}$
24. If the average of 3 consecutive ticket numbers is odd, then the sum of the least and greatest ticket numbers could be  
 A) 18                      B) 20                      C) 24                      D) 28



25. Eve counted to  $4^{60}$  by consecutive powers of 2, starting with  $2^1, 2^2, 2^3, \dots$ . How many powers of 2 did Eve count?
- A) 30                      B) 120                      C) 240                      D) 3600
26. How many even integers between 1 and 1 000 000 have digits that are all primes?
- A) 1365                      B) 3906                      C) 5400                      D) 19 530
27. If 6 identical machines can fill 80 bottles of soda in 12 seconds, how many seconds would it take 36 of the same machines to fill 240 bottles of soda?
- A) 6                              B) 12                              C) 18                              D) 24
28. Of my 100 favorite released songs, 42% were released after the year 2015 and 76% were released before the year 2017. What percent of my favorite songs were released in 2016?
- A) 18%                      B) 24%                      C) 34%                      D) 58%
29. (The number of positive even integers less than  $10^6$  that are perfect squares) : (the number of positive odd integers less than  $10^6$  that are perfect squares) =
- A) 1:1                      B) 2:1                      C) 499:500                      D) 999:1000
30. Of the following, which is a multiple of 4?
- A)  $2017^{2018} + 1$     B)  $2017^{2018} + 3$     C)  $2017^{2018} + 5$     D)  $2018^{2017} + 1$
31. If the sum of the measures of two angles of a parallelogram is 108 degrees, the sum of the measures of three of its angles could be
- A) 72 degrees              B) 162 degrees              C) 234 degrees              D) 252 degrees
32. Mr. Einstein hates repetition. He eats at a restaurant near his house once everyday. On the menu of this restaurant, there are 11 appetizers, 26 entrees, and 12 kinds of desserts. In addition, there are 12 wine selections offered. Mr. Einstein insists that everyday he eats a different meal combination that has never been served to him before. Each meal combination consists of one item from each of the four categories. For how many years can Mr. Einstein eat at this restaurant?
- A) 5                              B) 20                              C) 25                              D) Over 100
33. In the complete expansion of  $(x + 1)^4$ , what is the sum of the coefficients of the odd powers of  $x$ ?
- A) 4                              B) 6                              C) 8                              D) 10
34. A man starts with \$10000 and increases his wealth by 50 percent every three years. How much will he have in 12 years?
- A) \$30000                      B) \$50625                      C) \$70000                      D) None of the above
35. What is the sum of all positive two-digit integers which are divisible by both the sum and product of their digits?
- A) 36                              B) 54                              C) 72                              D) None of the above
36. If  $n$  is the smallest positive integer such that  $99n$  is the cube of an integer, and  $d$  is the sum of the digits of  $n$ , then  $d$  is
- A) 27                              B) 18                              C) 12                              D) 9
37. The area of my rectangle is 480. If my rectangle's length is 14 greater than its width, then its perimeter is
- A) 88                              B) 92                              C) 116                              D) 172
38. If  $\frac{4}{x} < 12$ , which of the following must always be true?
- A)  $x > 3$                       B)  $x > \frac{1}{3}$                       C)  $\frac{1}{x} < 3$                       D)  $\frac{1}{x} < \frac{1}{3}$



39. If  $x + y = a$  and  $xy = b$ , then what is the value of  $x^3 + y^3$  in terms of  $a$  and  $b$ ?

A)  $a^3 + 3ab$

B)  $a^3 - 3ab$

C)  $a^3 + b^3$

D)  $a^3 - b^3$

40. If I subtract the square of one integer from the square of another integer, then the difference could be

A) 386

B) 558

C) 768

D) 970

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七年级试卷答案

题号	1	2	3	4	5	6	7	8	9	10
答案	D	B	B	C	C	B	A	D	A	B
题号	11	12	13	14	15	16	17	18	19	20
答案	D	D	D	A	B	B	D	D	A	A
题号	21	22	23	24	25	26	27	28	29	30
答案	B	D	C	A	B	A	A	A	C	B
题号	31	32	33	34	35	36	37	38	39	40
答案	C	D	C	B	C	C	B	C	B	C